

Growth and phase velocity of self-modulated beam-driven plasma waves

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A long, relativistic charged particle beam propagating in a plasma is subject to the self-modulation instability. This instability is analyzed and the growth rate is calculated, including the phase relation. The phase velocity of the accelerating field is shown to be significantly less than the drive beam velocity. These results indicate that the energy gain of a plasma accelerator driven by a self-modulated beam will be severely limited by dephasing. In the long-beam, strongly-coupled regime, dephasing is reached in less than four e-foldings, independent of beam-plasma parameters.

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Plasma-based accelerators have attracted considerable attention owing to the ultrahigh field gradients sustainable in an electron plasma wave, enabling compact accelerators. The electric field amplitude of the electron plasma wave (space-charge oscillation) is on the order of $E_0 = cm_e\omega_p/e$, or $E_0[\text{V/m}] \simeq 96\sqrt{n_0[\text{cm}^{-3}]}$, where $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, n_0 is the ambient electron number density, m_e and e are the electron rest mass and charge, respectively, and c is the speed of light in vacuum. This field amplitude can be several orders of magnitude greater than conventional accelerators. Electron plasma waves with relativistic phase velocities may be excited by the nonlinear ponderomotive force of an intense laser [1] or the space-charge force of a charged particle beam, i.e., a plasma wakefield accelerator (PWFA) [2, 3]. In 2006, high quality 1 GeV electron beams were produced using using 40 TW laser pulses in cm-scale plasmas [4]. In 2007, a 42 GeV electron beam in a meter-long plasma was used to double the energy of a small fraction of electrons on the beam tail by the plasma wave excited by the beam head [5]. These experimental successes have resulted in further interest in the development of plasma-based acceleration as a basis for future linear colliders [6, 7].

It has recently been proposed to drive a plasma accelerator with a highly relativistic proton beam, such as those available at CERN (European Organization for Nuclear Research) [8, 9]. In general, exciting plasma waves requires a drive beam density profile with frequency components at the plasma frequency, i.e., a beam density longitudinal scale length on the order of the plasma wavelength $\lambda_p = 2\pi/k_p = 2\pi c/\omega_p$, or $\lambda_p[\mu\text{m}] = 3.3 \times 10^{10}/\sqrt{n[\text{cm}^{-3}]}$. Compact, high-gradient accelerators require high plasma density, and therefore require short drive beams, e.g., $\lambda_p \sim 100 \mu\text{m}$ for $n_0 \sim 10^{17} \text{cm}^{-3}$. Generating short proton beams (or proton beams with spatial structure at λ_p) is challenging, and it has been proposed to rely on a beam-plasma instability to modulate the beam at λ_p , driving a large amplitude plasma wave [10]. The self-modulation of the beam occurs through coupling of the transverse wakefield with

the beam radius evolution. Periodic regions of focusing and defocusing modulate the beam density at λ_p , driving a larger plasma density modulation that further focuses the beam periodically. This is somewhat similar to the self-modulation instability that occurs for long laser pulses [11]. For beams long compared to λ_p , where self-modulation occurs, the instability is enabled by the drive beam dynamics, and therefore the wakefield properties will be strongly affected by the drive beam dynamics.

An important quantity characterizing the performance of a plasma accelerator is the phase velocity v_p of the plasma wave. For $v_p < c$, a highly relativistic electron will outrun the plasma wave and phase slip from the accelerating to the decelerating phase region of the plasma wave. This limits the electron energy gain to $\Delta W \sim \gamma_p^2(E_z/E_0)m_e c^2$ after acceleration over a dephasing length $L_d \sim \gamma_p^2 \lambda_p$, where E_z is the electric field amplitude of the plasma wave and $\gamma_p = (1 - v_p^2/c^2)^{-1/2}$. For a plasma accelerator driven by a short ($< \lambda_p$) intense laser pulse, v_p can be relatively low ($\gamma_p \sim 10 - 100$) and dephasing can limit the energy gain [12]. For a PWFA driven by a short ($< \lambda_p$) highly-relativistic beam, v_p can be sufficiently high so that dephasing is not an issue.

In this Letter we calculate the self-modulation of particle beams in plasma, including the properties of the excited plasma wave. In particular, we show that the phase velocity of the plasma wave excited by self-modulation is greatly reduced from the velocity of the drive beam. The phase velocity is determined by the growth of the instability and the beam-plasma dynamics. A similar effect occurs in self-modulated laser-driven plasma waves [13, 14]. Analytic solutions for the growth rate and phase velocity in the long beam regime are derived and compared to numerical solutions of the envelope equation for the particle beam. Owing to the low phase velocity of the plasma wave, the maximum energy gain in such a self-modulated beam-driven accelerator will be severely limited by dephasing.

The wake generated by a relativistic particle beam driver moving through a plasma can be calculated using the cold plasma fluid and Maxwell equations. Here we

consider a drive beam consisting of particles with charge $\mp e$ and mass M_b . In the linear wake regime, the normalized density perturbation $\delta n/n_0 = (n - n_0)/n_0 < 1$ driven by a beam with density $n_b < n_0$ is

$$(\partial_\zeta^2 + k_p^2) \delta n/n_0 = \mp k_p^2 n_b/n_0, \quad (1)$$

where the \mp corresponds to a negatively/positively charged particle beam. A highly relativistic beam is assumed with Lorentz factor $\gamma = (1 - \beta_b^2)^{-1/2} \gg 1$, and the quasi-static approximation is taken such that the plasma fluid quantities are functions of the co-moving variable $\zeta = z - \beta_b t$. The beam-driven longitudinal electric field E_z and transverse fields E_r and B_θ are [15]

$$(\nabla_\perp^2 - k_p^2) E_z/E_0 = -k_p \partial_\zeta \delta n/n_0, \quad (2)$$

$$(\nabla_\perp^2 - k_p^2) (E_r - B_\theta)/E_0 = -k_p \partial_r \delta n/n_0. \quad (3)$$

The transverse beam-driven wakefield Eq. (3) is coupled to the envelope equation for the beam [16]

$$\frac{d^2 R}{dz^2} - \frac{\epsilon_n^2}{4\gamma^2 R^3} = \mp \frac{1}{\gamma R} \frac{m_e}{M_b} \langle k_p r (E_r - B_\theta)/E_0 \rangle, \quad (4)$$

where $R = \langle r^2 \rangle^{1/2}$ is the rms beam size, $\epsilon_n = \gamma[\langle r^2 \rangle \langle (dr/dz)^2 \rangle - \langle r dr/dz \rangle^2]^{1/2}/2$ is the normalized transverse emittance in cylindrical geometry, and the brackets indicate an average over the transverse beam distribution.

For simplicity, in the following we consider a beam with a flat-top radial profile, $n_b = [n_{b0} r_{b0}^2/r_b^2] f(\zeta) \Theta(r - r_b)$, where f is the normalized longitudinal profile, Θ is the Heaviside function, $r_b(\zeta, z)$ is the beam radius, and $r_{b0} = r_b(\zeta, z = 0)$ is the initial beam radius. For a flat-top radial profile, Eqs. (1) and (3) have the solution

$$(E_r - B_\theta)/E_0 = \pm (n_{b0}/n_0) k_p^2 r_{b0}^2 I_1(k_p r) \times \int_\infty^\zeta d\zeta' \sin[k_p(\zeta - \zeta')] f(\zeta') K_1(k_p r_b(\zeta'))/r_b(\zeta'), \quad (5)$$

for $r \leq r_b$, assuming the initial radius r_{b0} is independent of ζ . Here I_m and K_m are the modified Bessel functions. Using Eqs. (4) and (5), the envelope equation for the beam radius $r_b(\zeta, z) = \sqrt{2}R$ at any slice ζ is

$$\frac{d^2 r_b}{dz^2} - \frac{\epsilon_n^2}{\gamma^2 r_b^3} = - \frac{4k_b^2 r_{b0}^2 I_2(k_p r_b)}{\gamma r_b} \times \int_\infty^\zeta d\zeta' \sin[k_p(\zeta - \zeta')] f(\zeta') K_1(k_p r_b(\zeta'))/r_b(\zeta'), \quad (6)$$

where $k_b^2 = 4\pi n_{b0} e^2/M_b$ is plasma wavenumber of the beam. Equation (6) describes the coupled beam evolution and wakefield excitation.

Consider a long beam compared to the plasma wavelength, where the variation in the longitudinal profile may be neglected $f(\zeta) \simeq 1$, propagating in a plasma

with a perturbation at the plasma wavelength. In the following we will decompose the plasma and beam quantities such that $Q = Q_0 + Q_1$, where the ‘0’ subscripts indicate the long beam solution and the ‘1’ subscripts indicate the perturbation. The initial perturbation or instability seed may be due to excitation of a plasma wave from the head of the beam, fluctuations in the beam or plasma, or from a plasma wave externally excited (e.g., by a short-pulse laser). Using Eqs. (1)–(4), the unperturbed long beam solution is $(\delta n)_0 = -n_b$, $(E_z)_0 = 0$, $(E_r - B_\theta)_0 = \pm E_0(n_{b0}/n_0) k_p r_0 K_1(k_p r_0) I_1(k_p r) (r_{b0}/r_0)^2$ for $r \leq r_0$, and the beam radius evolves as

$$\frac{d^2 r_0}{dz^2} - \frac{\epsilon_n^2}{\gamma^2 r_0^3} + \frac{4k_b^2 r_{b0}^2}{\gamma k_p r_0^2} K_1(k_p r_0) I_2(k_p r_0) = 0. \quad (7)$$

We will assume that the beam is initially in the long beam equilibrium $r_{b0} = r_0 = r_{eq}$, such that $d^2 r_0/dz^2 = 0$, and r_{eq} is given by $\epsilon_n^2 k_p = 4\gamma k_b^2 r_{eq}^3 K_1(k_p r_{eq}) I_2(k_p r_{eq})$. For a narrow beam, $k_p r_{eq} \ll 1$, the equilibrium beam radius is $r_{eq} = [2\epsilon_n^2/k_b^2 \gamma]^{1/4}$.

Assuming a small perturbation about this equilibrium, $r_b = r_0 + r_1$ with $|r_1/r_0| \ll 1$, and expanding Eq. (6) yields the evolution of the beam radius perturbation

$$\left(\frac{d^2}{d\hat{z}^2} + 4\kappa^2 \right) r_1 = 2\nu \int_\infty^\zeta d\hat{\zeta}' \sin(\hat{\zeta} - \hat{\zeta}') r_1(\hat{\zeta}'), \quad (8)$$

with the constants

$$\kappa^2 = 2K_1(k_p r_0) \left[4 \frac{I_2(k_p r_0)}{k_p r_0} + I_3(k_p r_0) \right], \quad (9)$$

and $\nu = 4I_2(k_p r_0) K_2(k_p r_0)$, and the normalized variables $\hat{\zeta} = k_p \zeta$ and $\hat{z} = k_b z/(2\gamma)^{1/2}$. In the limit of a narrow beam $k_p r_0 \ll 1$, $\nu \simeq 1 - (k_p r_0)^2/6$, and $\kappa^2 \simeq 1 + (k_p r_0)^2 [C_\gamma - 1/4 + \ln(k_p r_0/2)]/2$, where $C_\gamma \simeq 0.577$ is the Euler-Mascheroni constant. Equation (8) may be analyzed in several regimes. The most relevant regime for plasma accelerators based on self-modulated drive beams is the strongly-coupled (or long-beam, early-time) regime valid for $\hat{\zeta} \gg \hat{z}$.

Applying the linear plasma wave operator to Eq. (8) yields

$$(\partial_\zeta^2 + 1)(\partial_{\hat{z}}^2 + 4\kappa^2) r_1 = 2\nu r_1. \quad (10)$$

Consider a slowly varying envelope, such that $r_1 = \hat{r} \exp(ik_p \zeta)/2 + \text{c.c.}$ with $|\partial_\zeta \hat{r}| \ll |k_p \hat{r}|$, and assume the strongly-coupled regime where the growth length of the instability is short compared to $\gamma^{1/2} k_b^{-1}$, such that $|\partial_{\hat{z}} \hat{r}| \gg 2\kappa |\hat{r}|$. In this regime Eq. (10) becomes

$$(\partial_{\hat{z}}^2 + i\nu) \hat{r} = 0, \quad (11)$$

which describes the evolution of the slowly varying amplitude of the beam radius perturbation and may be solved

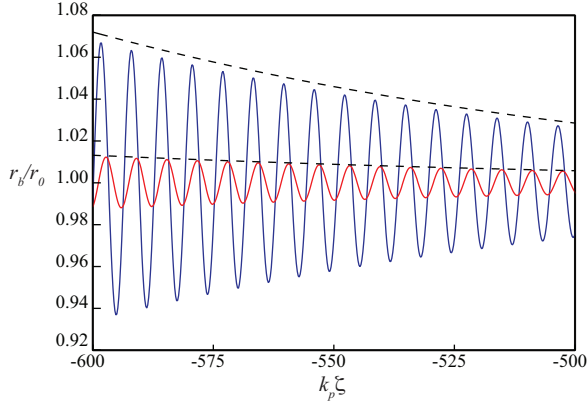


FIG. 1. (Color online) Beam radius modulation r_b/r_0 vs $k_p \zeta$ with beam-plasma parameters $n_b/n_0 = 0.008$, $\gamma = 107$, and $k_p r_0 = 1$ (and $r_{b0} = r_0 = r_{eq}$), obtained from numerical solution of Eq. (6), at $k_p z = 8000$ (red curve) and $k_p z = 9500$ (blue curve). Dashed curves are the envelope of the asymptotic linear solution Eq. (13).

using standard Laplace transform techniques. With the initial conditions $\hat{r}(z, \zeta = 0) = \delta r \Theta(z)$, $\hat{r}(z = 0, \zeta) = \delta r$, and $\partial_z \hat{r}(z = 0, \zeta) = 0$, the solution to Eq. (11) can be expressed as

$$\hat{r}/\delta r = \sum_{n=0}^{\infty} \frac{(i\nu|\hat{\zeta}|\hat{z}^2)^n}{n!(2n)!} = {}_0F_2(; \{1/2, 1\}; i\nu|\hat{\zeta}|\hat{z}^2/4), \quad (12)$$

where ${}_qF_p$ is the generalized hypergeometric function. The solution to Eq. (11) may also be evaluated asymptotically and has the form

$$r_1 = \delta r \frac{3^{1/4}}{(8\pi)^{1/2}} N^{-1/2} e^N \cos(k_p \zeta + N/\sqrt{3} - \pi/12), \quad (13)$$

where the number of e-foldings is

$$N = \frac{3^{3/2}}{4} \left(\nu \frac{n_b m_e}{n_0 M_b \gamma} k_p^3 |\zeta| z^2 \right)^{1/3}. \quad (14)$$

Note that growth Eq. (14) [and the beam envelope equation, Eq. (6)] differ from that found in Ref. [10].

Figure 1 shows the beam radius modulation $r_b/r_0 = 1 + r_1$ versus $k_p \zeta$, after propagating $k_p z = 8000$ (red curve) and $k_p z = 9500$ (blue curve), obtained from numerical solution of Eq. (6) for a beam initially in equilibrium $r_{b0} = r_0 = r_{eq}$ with beam-plasma parameters $n_b/n_0 = 0.008$, $\gamma = 107$, and $k_p r_0 = 1$. The dashed curves are the envelope of the linear asymptotic solution Eq. (13). Figure 1 shows the growth versus distance behind the head of the beam (at $k_p \zeta = 0$) and versus propagation distance. Also shown is the shift in phase of the modulation versus propagation distance, resulting in a reduced phase velocity, as discussed below.

The above solution Eq. (12) assumed $|k_p \hat{r}| \gg |\partial_\zeta \hat{r}|$, or $1 \gg |k_p^{-1}(\partial_\zeta N)|$. This condition may be expressed as

$$1 \gg \frac{3^{3/2}}{2^6} \left(\frac{k_b^2 \nu}{k_p^2 \gamma} \right) \left(\frac{z}{|\zeta|} \right)^2, \quad (15)$$

or $\hat{\zeta} \gg \hat{z}$, which will be satisfied for long beams sufficiently early in the beam propagation. It was also assumed that $|\partial_z \hat{r}| \gg 2\kappa|\hat{r}|$, which is satisfied provided Eq. (15) is satisfied. The above analysis is also based on linear theory, and nonlinear effects (i.e., when $r_1 \sim r_0$ or $E_z \sim E_0$) may saturate the instability.

The beam radius perturbation $r_1 = \hat{r} \exp(ik_p \zeta)/2 + \text{c.c.}$ modulates beam density $n_b \simeq n_b(r_0)(1 - 2r_1/r_0)$. This beam density modulation drives a modulation in the electron plasma density $\hat{n} \exp(ik_p \zeta)/2 + \text{c.c.}$, via Eq. (1), i.e., $\partial_\zeta \hat{n} \simeq \mp i k_p n_b(r_0) \hat{r}/r_0$. The plasma density modulation drives the accelerating wakefield $E_z/E_0 = \hat{E}_z \exp(ik_p \zeta)/2 + \text{c.c.}$, via Eq. (2), i.e., $(\nabla_\perp^2 - k_p^2) \partial_\zeta \hat{E}_z = \mp k_p^3 [n_b(r_0)/n_0] \hat{r}/r_0$. For the same initial conditions as above, the series solution for the accelerating wakefield in the long-beam regime is

$$\hat{E}_z = \mp H_R(r, r_0) \frac{n_{b0}}{n_0} \frac{\delta r}{r_0} |\hat{\zeta}| \sum_{n=0}^{\infty} \frac{(i\nu|\hat{\zeta}|\hat{z}^2)^n}{(n+1)!(2n)!}, \quad (16)$$

and the sum may be expressed as the hypergeometric function ${}_0F_2(; \{1/2, 2\}; i\nu|\hat{\zeta}|\hat{z}^2/4)$. Here $H_R(r, r_0) = 1 - k_p r_0 K_1(k_p r_0) I_0(k_p r)$ for $r \leq r_0$ and $k_p r_0 I_1(k_p r_0) K_0(k_p r)$ for $r > r_0$. In the asymptotic limit, $E_z/E_0(z = 0) \simeq 3^{7/4} (32\pi)^{-1/2} N^{-3/2} \exp(N) \cos(\psi)$, where the number of e-foldings of growth of the accelerating wake is given by Eq. (14) and the phase is

$$\psi = k_p \zeta - \frac{\pi}{4} + \frac{3}{4} \left(\nu \frac{k_b^2 k_p}{\gamma} |\zeta| z^2 \right)^{1/3}. \quad (17)$$

The phase velocity of the accelerating wake is given by $\beta_p = -\partial_t \psi / \partial_z \psi = \partial_\zeta \psi / (\partial_\zeta + \partial_z) \psi \simeq 1 - \partial_z \psi / \partial_\zeta \psi$. In this regime, i.e., satisfying Eq. (15), $\beta_p \simeq 1 - k_p^{-1} \partial_z \psi$. Using the phase Eq. (17), the phase velocity is $\beta_p = 1 - (2/3^{3/2})(N/k_p z)$. The phase velocity of the self-modulated beam-driven wakefield is less than the beam velocity $\beta_b \simeq 1$, varies along the beam ζ and during propagation z . Asymptotically, the Lorentz factor of the phase velocity is

$$\gamma_p = \left(\frac{\gamma n_0 M_b}{\nu n_{b0} m_e} \frac{z}{|\zeta|} \right)^{1/6} \quad (18)$$

in the strongly-coupled, long-beam regime. Note that, behind the modulated beam the phase velocity is given by Eq. (18) with $|\zeta| = L_b$, where L_b is the bunch length.

Figure 2 shows the normalized Lorentz factor of the phase velocity of the accelerating wakefield $\gamma_p [\nu (k_b/k_p)^2 |\hat{\zeta}|/\gamma]^{-1/4}$ versus normalized propagation

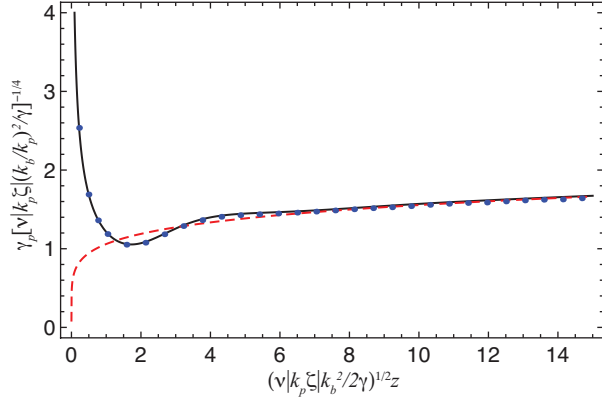


FIG. 2. (Color online) Normalized phase velocity of accelerating wakefield $\gamma_p [\nu |k_b/k_p|^2 |\zeta|/\gamma]^{-1/4}$ vs. normalized propagation distance $(\nu |\zeta|)^{1/2} \hat{z}$ in the long-beam regime: solid (black) curve is the series solution Eq. (16), dashed (red) curve is the asymptotic solution Eq. (18), and (blue) dots are from the numerical solution of the envelope equation Eq. (6).

distance $(\nu |\zeta|)^{1/2} \hat{z}$. The solid curve in Fig. 2 is obtained from the series solution Eq. (16), $\beta_p = 1 - k_p^{-1} \partial_z [\arctan(\Im \hat{E}_z / \Re \hat{E}_z)]$, the dashed curve is the asymptotic solution Eq. (18), and the dots are from the numerical solution (with the parameters $\gamma = 107$, $n_{b0}/n_0 = 0.008$, $k_p r_0 = 1$, and $k_p L_b = 600$) of the envelope equation Eq. (6). Figure 2 indicates that there is a minimum phase velocity. The minimum phase velocity can be estimated by using the series solution Eq. (16). The minimum phase velocity occurs at $(\nu |\zeta|)^{1/2} \hat{z} \simeq 1.72$, with

$$\gamma_{\min} \simeq 1.06 \left(\frac{\gamma n_0 M_b}{\nu n_{b0} m_e k_p |\zeta|} \right)^{1/4}. \quad (19)$$

As shown in Fig. 2, after reaching γ_{\min} , the phase velocity grows slowly as the beam propagates $\gamma_p \propto z^{1/6}$ [cf. Eq. (18)]. For example, consider a wake driven by a 100 GeV proton beam ($\gamma = 107$), with $r_0 = 180 \mu\text{m}$, $L_b = 10 \text{ cm}$, and 10^{11} particles. Operating at $n_0 = 10^{15} \text{ cm}^{-3}$, corresponds to $n_{b0}/n_0 = 0.008$, $E_0 = 3 \text{ GV/m}$, $k_p r_0 = 1.0$, $k_p L_b = 600$, and $\nu = 0.88$. For this example, $\gamma_{\min} \simeq 15$ behind the drive beam after $z \simeq 8.5 \text{ cm}$ (i.e., $\approx 500 \lambda_p$) of propagation.

With the phase velocity of the self-modulated wake determined, the dephasing length may be calculated. For a linear wake, the dephasing length is the propagation distance required for an ultra-relativistic particle $\beta_b \simeq 1$ to slip $\lambda_p/4$ (or a wake phase of $\pi/2$) with respect to the plasma wave. Assuming the phase velocity is well-approximated by the asymptotic solution in the strongly-coupled regime Eq. (18), the dephasing length is $L_d = (2\pi/3)^{3/2} (\nu k_b^2 k_p |\zeta_i|/\gamma)^{-1/2}$. Including the early

time response via Eq. (16), the dephasing length is

$$L_d \simeq 4.9 (\nu k_b^2 k_p |\zeta_i|/\gamma)^{-1/2}, \quad (20)$$

where ζ_i is the injection position of the witness bunch (e.g., initially at a peak of the accelerating field). For a witness bunch injected behind the drive beam, $\zeta_i = L_b$. This reduced dephasing length will greatly limit the energy gain of a witness electron beam trailing the drive bunch. For example the number of e-foldings of the self-modulational instability that have occurred at the dephasing length Eq. (20) is $N(z = L_d, \zeta_i) \simeq 3.8$. Note that the number of e-foldings at a dephasing length $N(z = L_d)$ is independent of injection location and the beam-plasma parameters. Improved efficiency may be possible by tapering the plasma density, i.e., increasing the background plasma density to reduce the plasma wavelength, thereby increasing the phase velocity [17], although variation of the density may affect the instability growth. Alternatively the accelerator may use a staged approach, where a long plasma region self-modulates the drive beam, followed by a second stage where a witness bunch would be injected following the modulated drive beam. Such a two-staged approach could potentially be limited by the hose (or transverse two-stream) instability [18], which grows in the long beam limit with a comparable growth rate $\sim N$. This implies that to drive large amplitude accelerating fields via the self-modulational instability without hosing requires strongly seeding the instability. One possibility is to use a beam with a fast rise in the current profile [10]. Another possibility to seed the modulation is via an intense short-pulse laser.

The long-beam, early-time regime described above will be valid for $\hat{z} \ll \hat{\zeta}$. After sufficiently long propagation distances, or for sufficiently short beams, the instability may enter a weakly-coupled regime where the instability growth length is long compared to $\gamma^{1/2}/k_b$. The instability will transition to the weakly-coupled regime after a propagation distance approximately $\hat{z} \sim \hat{\zeta}$, or, using Eq. (14), after approximately $N \sim k_p \zeta$. For long beams $k_p \zeta \gg 1$, nonlinear effects will typically appear before the instability enters the weakly-coupled regime.

In this Letter we have calculated the beam self-modulation instability growth rate, in the long-beam regime, including the phase dependence. The phase velocity of the accelerating wakefield was calculated and shown to be significantly less than the drive beam velocity. The dephasing length was calculated, and, in the strongly-coupled regime, a witness beam will reach dephasing in less than four e-foldings, independent of beam-plasma parameters. This indicates that the energy gain in a plasma accelerator driven by a self-modulated PWFA will be limited by dephasing.

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- [1] E. Esarey, C. B. Schroeder, and W. P. Leemans, Rev. Mod. Phys. **81**, 1229 (2009).
 - [2] P. Chen *et al.*, Phys. Rev. Lett. **54**, 693 (1985).
 - [3] J. B. Rosenzweig, Phys. Rev. A **38**, 3634 (1988).
 - [4] W. P. Leemans *et al.*, Nature Phys. **2**, 696 (2006).
 - [5] I. Blumenfeld *et al.*, Nature **445**, 741 (2007).
 - [6] A. Seryi *et al.*, in *Proceedings of PAC09* (JACoW, Vancouver, BC, 2009).
 - [7] C. B. Schroeder *et al.*, Phys. Rev. ST Accel. Beams **13**, 101301 (2010).
 - [8] A. Caldwell *et al.*, Nature Phys. **5**, 363 (2009).
 - [9] K. V. Lotov, Phys. Rev. ST Accel. Beams **13**, 041301 (2010).
 - [10] N. Kumar, A. Pukhov, and K. Lotov, Phys. Rev. Lett. **104**, 255003 (2010).
 - [11] E. Esarey, J. Krall, and P. Sprangle, Phys. Rev. Lett. **72**, 2887 (1994).
 - [12] C. B. Schroeder *et al.*, Phys. Rev. Lett. **106**, 135002 (2011).
 - [13] N. E. Andreev *et al.*, IEEE Trans. Plasma Sci. **24**, 363 (1996).
 - [14] W. P. Leemans *et al.*, IEEE Trans. Plasma Sci. **24**, 331 (1996).
 - [15] R. Keinigs and M. E. Jones, Phys. Fluids **30**, 252 (1987).
 - [16] M. Reiser, *Theory and Design of Charged Particle Beams*, 2nd ed. (Wiley-VCH, Weinheim, 2008).
 - [17] T. Katsouleas, Phys. Rev. A **33**, 2056 (1986).
 - [18] D. H. Whittum, Phys. Plasmas **5**, 4432 (1993).